HOW FAST/FAR DOES FLY LINE FALL?

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March 2003

This report summarizes a simple model for the "free fall" dynamics of a length of fly line. The line is assumed to remain horizontal and the only forces acting on it are its weight and air drag. Under these two forces, the line accelerates downwards, and this acceleration decreases as the line approaches a terminal velocity.

A sketch of the fly line below shows the weight force $\vec{W}$ and the drag force $\vec{D}$. This sketch also defines the positive direction for the fly line displacement *y,* velocity *v*, and acceleration *a* as downward.

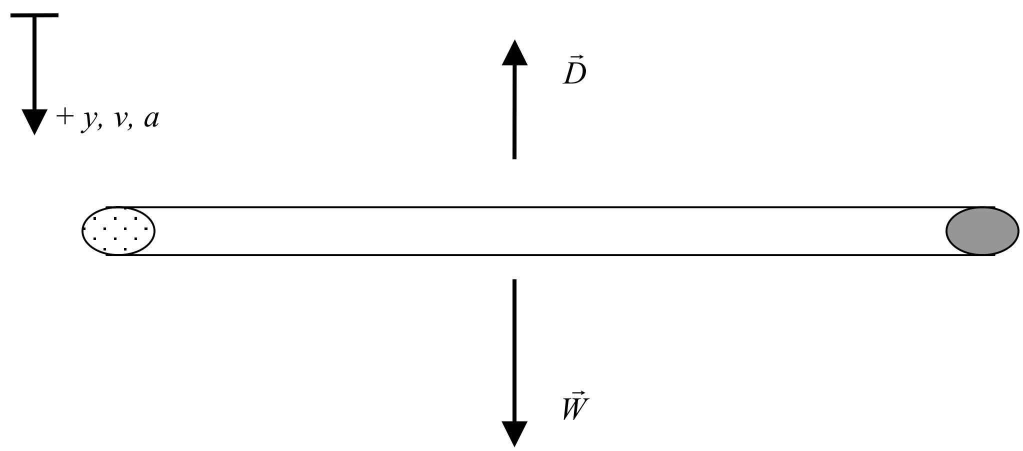


Figure 1. Definition of freely falling fly line subject to weight and air drag.

Define the following parameters for the fly line

ρ_l	density of fly line
d	diameter of fly line
L	length of fly line sample

Then the weight of the sample is given by $W = \rho_l \dfrac{\pi d_l^2}{4} Lg$ and it is directed downwards where g denotes the gravitational constant.

Define the following parameters for the form drag

$C_{dn} = 1$	drag coefficient for form drag (value of 1 is good approximation)
ρ_a	density of air

Then the form drag is given by $D = \frac{1}{2}\rho_a d_l L C_{dn} v^2$ and it is directed upwards.

Apply Newton's law ($W - D = ma$) in the vertical direction to arrive at

$$\rho_l \frac{\pi d_l^2}{4} Lg - \frac{1}{2}\rho_a d_l L C_{dn} v^2 = (\rho_l \frac{\pi d_l^2}{4} L)a \tag{1.1}$$

Notice that the length L cancels and therefore the result is independent of the length of the fly line sample as expected. Equation (1.1) can be written in the equivalent form

$$\frac{dv}{dt} = g - bv^2 \tag{1.2}$$

where the constant

$$b = \frac{2}{\pi}\frac{\rho_a}{\rho_l}\frac{C_{dn}}{d_l} \tag{1.3}$$

Integrating Eq. (1.2) using the initial conditions (*t=0,v=0)* for starting at rest, and then solving for v yields the result

$$v(t) = \sqrt{\frac{g}{b}\left[\frac{e^{2t\sqrt{gb}} - 1}{e^{2t\sqrt{gb}} + 1}\right]} \tag{1.4}$$

that can be used to compute the fly line speed as a function of time after release.

Notice that the terminal velocity of the fly line is given by

$$v_t = \sqrt{\frac{g}{b}} = \sqrt{\frac{g\pi}{2}\frac{\rho_l}{\rho_a}\frac{d_l}{C_{dn}}} \tag{1.5}$$

We can also compute the distance the fly line has fallen after first noting that $a = v\frac{dv}{dy}$.

Thus, Eq. (1.2) can be re-written as

$$v\frac{dv}{dy} = g - bv^2 \tag{1.6}$$

and integrating this result using the initial conditions (*v=0, y=0*) leads to

$$y = \frac{1}{2b}\ln\left[\frac{g}{g - bv^2}\right] \tag{1.7}$$

that can be used to compute the distance fallen as a function of speed.

As a summary, here are the steps needed to compute the speed and distance fallen after release.

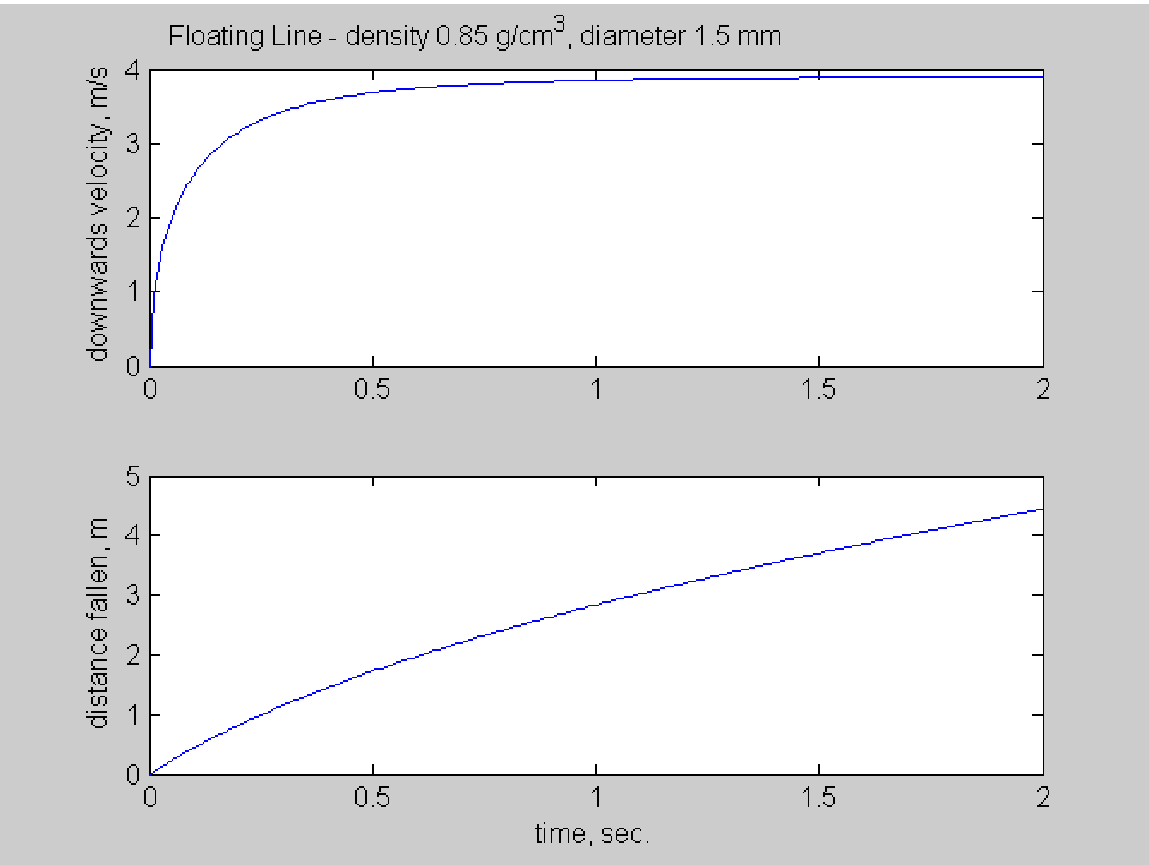
1. Compute the fly line constant “b” using Eq. (1.3).
2. Compute the fly line speed as a function of time using Eq. (1.4).
3. Use the computed speed to compute the corresponding distance using Eq. (1.7).

Example: Floating Line and Sinking Line of Equal Weight

Suppose we have two lines with the *same weight* but *different diameters*. One line is a floater with density 0.85 g/cm^3 and diameter 1.5 mm. The other line is a sinker with density 2.5 g/cm^3 and diameter of 0.875 mm. Samples of equal length will have equal weight ($\rho_l d_l^2$ is the same). We now drop equal lengths of these lines from rest and from horizontal. They will fall under the action of gravity and this free fall will be partially resisted by form drag. The plots below show the velocity and distance fallen during a two-second time interval after release for each line.

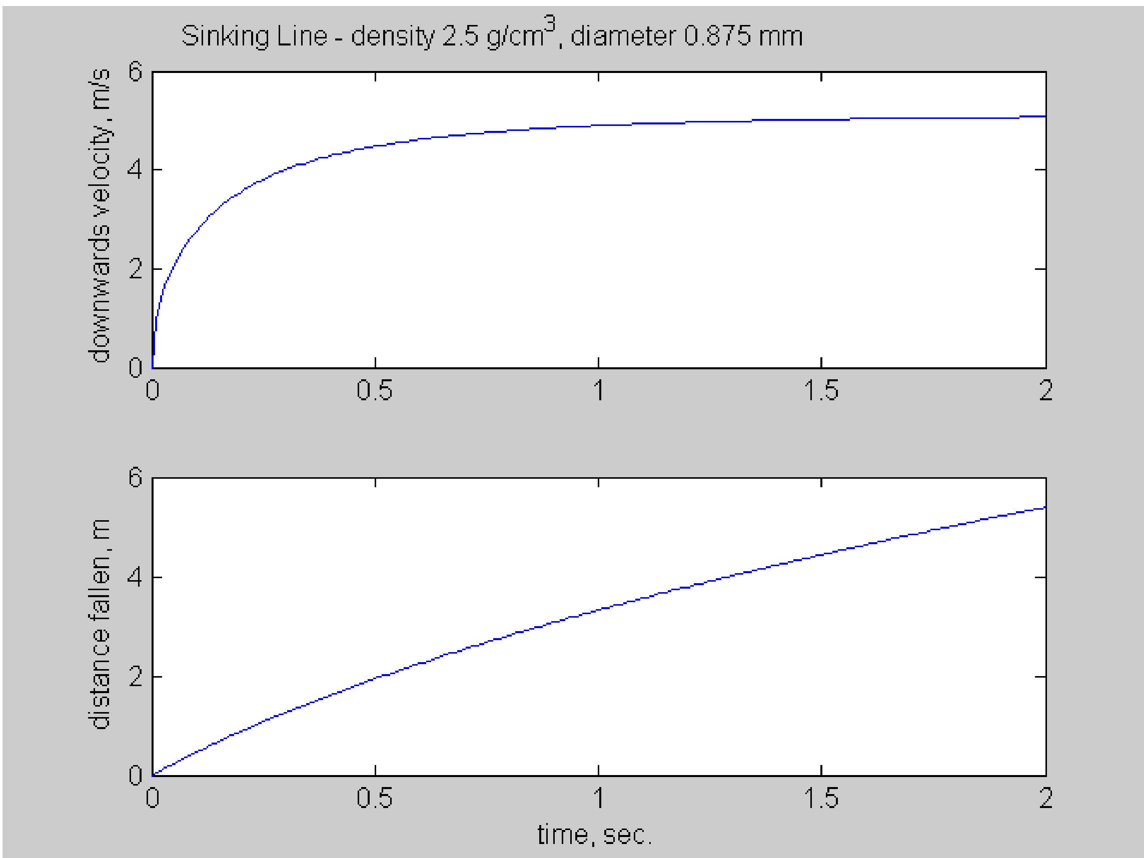
Results for Floating Line.

The terminal velocity in this case is 3.90 m/s. After 1 second, this line has fallen a distance of 2.84 m and has a velocity of 3.85 m/s (thus, 98.7% of the terminal velocity has already been achieved). If there were no form drag, this line would achieve a velocity of 9.81 m/s after 1 second and the velocity will continue to increase (linearly with time). Moreover, it would fall a distance of 4.91 m in 1 second. Thus, form drag plays a very large role.

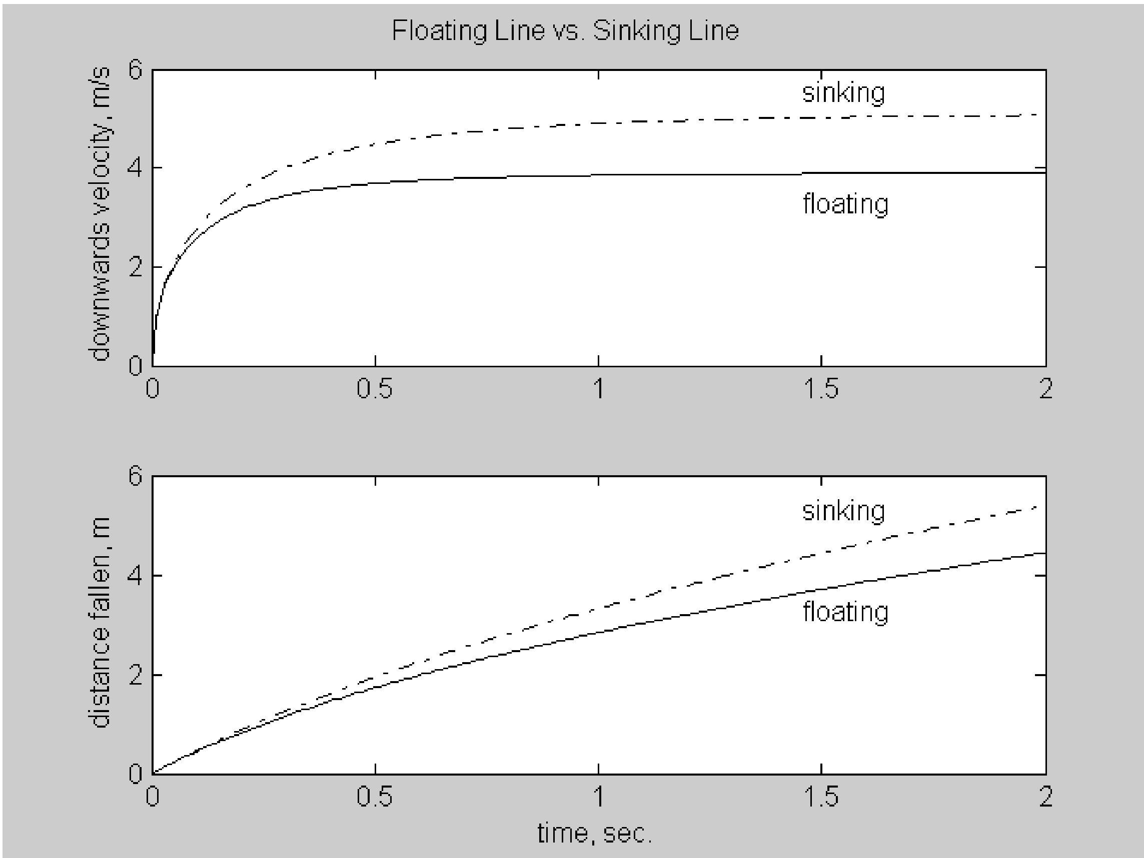


Results for Sinking Line

The terminal velocity in this case is 5.11 m/s. After 1 second, this line has fallen a distance of 3.32 m and has a velocity of 4.90 m/s (thus, 95.8% of the terminal velocity has already been achieved). If there were no form drag, this line would achieve a velocity of 9.81 m/s after 1 second and the velocity will continue to increase (linearly with time). Moreover, it would fall a distance of 4.91 m in 1 second. Thus, form drag plays a very large role.



The following figure makes for a quick comparison of the behavior of these two lines. Recognize that a typical fly rod would be close to 3m long on the vertical scale below for "distance fallen".



In all of these results, it is important to remember that the model used does not account for the tension in the line and that line tension will alter things. Consider the lower leg of the loop. As the loop falls a bit, the lower leg forms a gentle downward slope from the rod tip to the loop bottom. The tension is directed along this slope and therefore there is a small and upwards component of the tension force where the line attaches to the rod tip. This tension component further serves to slow the fall of the line.